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Sub-discrete Structure

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Ans No. 1

given statement: $1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \dots + n \cdot n! = (n+1)! - 1$

let check for value $n=1$

$$= (1+1)! - 1$$

$$= 2 - 1$$

$$= 1$$

now lets check for value $n=2$

$$= (2+1)! - 1$$

$$= 6 - 1$$

$$= 5$$

so this is true for $n=1, 2$ now let assume that it is true for $n=k$ and $n=k+1$ so $n=k+1$

$$= (k+1+1)! - 1$$

$$= (k+2)! - 1$$

and $n = k$

$$= (k+1)! - 1$$

now, $1 \cdot 1! + 2 \cdot 2! + \dots + k \cdot k! +$ (adding both side $(k+1)(k-1)!$

$$\Rightarrow (k+1)! + (k+1)(k+1)! - 1$$

$$= (k+1)! (k+1+1) - 1$$

$$= (k+1)! (k+2) - 1$$

$$= (k+2)! - 1 \quad \text{which is true for } k+1 \text{ therefore}$$

the statement is true for all positive integers.

(i) one particular player is always included.

we have total no. of members is 15 and we need a team of 11 players.

so, 1 player is always included. we need to get combination of ${}^{14}C_{10}$

$${}^{14}C_{10} = \frac{14!}{10! 4!} = \frac{14 \times 13 \times 12 \times 11 \times 10!}{4 \times 3 \times 2 \times 1}$$

there are 1001 ways to select.

(ii) if two such players always been include than both two players will act as one when getting combination.

$${}^{13}C_9 = \frac{13!}{9! 4!}$$
$$= \frac{13 \times 12 \times 11 \times 10}{4 \times 3 \times 2}$$

$$= 13 \times 11 \times 5$$

$$= 715$$

there are 715 ways to select.

given relation

$$R = \{ (a, b) \in R \mid a - b \leq 3 \}$$

① so for reflexive it should have $(a, a) \in R$
if, $a = b$

$$a - a \leq 3$$

$$0 \leq 3$$

which is true. therefore R is reflexive.

② now for symmetric $(b, a) \in R$ and $(a, b) \in R$.
so for (b, a) .

$$-(a - b) \geq -3$$

$$b - a \geq -3$$

which is true since a will always be greater than b .

③ now to check transitive we have to create new relation
such that $a, b, c \in R$

$$(a, b) \in R, (b, c) \in R, (a, c) \in R$$

$$a - b \leq 3, b - c \leq 3.$$

on adding this

$$a - b + b - c \leq 3 + 3$$

$$a - c \leq 6$$

which is true because a is not getting higher than 3

4) it can't be antisymmetric due $a \neq b \in R$

the length of string is 6

, so total no. of combinations with 1 is $6! = 720$

now if only 1, 1, 1 are in front then we have to calculate how many combination are there with having same 1.

$$\Rightarrow \frac{6!}{4!} = \frac{6 \times 5 \times 4!}{4!} = 30$$

and we have to treat them as one bit string (1, 1, 1)

total combination will become $3! = 6$

therefore total no. of combination excluding substring

$$\text{is } = 720 - 30 + 6$$

$$= 696$$

\therefore there are 696 ways.